EX NAVODAYAN FOUNDATION
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Full Syllabus

JEE-Main

Paper-6

Test Date: **M.M:300**

TEST INSTRUCTIONS

1. The test is of **3 hours** duration.
2. The test booklet consists of **75 questions**.
3. The maximum marks are **300**.
4. All questions are compulsory.
5. There are three parts in the questions paper consisting of Physics, Chemistry and Mathematics having **25 questions in each part**.

Each Parts Contains –

- 20 multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct. All questions are carrying **+4 marks** for right answer and **-1 mark** for wrong answer.
- 05 questions with answer as **numerical value** all questions are carrying **+4 marks** for right answer and **–1 marks** for wrong answers.

Name of the Candidate (in Capital Letter): _____

Registration No. _____

Invigilator Signature

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Physics

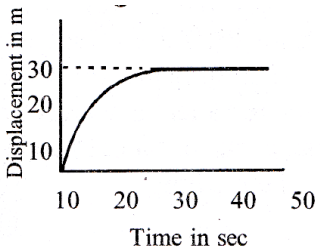
(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. The resistance R of a wire is given by the relation $R = \frac{\rho \ell}{\pi r^2}$. Percentage error in the measurement of ρ, ℓ and r is 1%, 2% and 3% respectively. Then the percentage error in the measurement of R is

   (a) 6 (b) 9 (c) 8 (d) 10

2. The displacement of a particle as a function of time is shown in figure. It indicates that



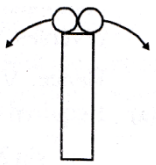
   (a) the velocity of the particle is constant throughout

   (b) the acceleration of the particle is constant throughout

   (c) the particle starts with a constant velocity and is accelerated

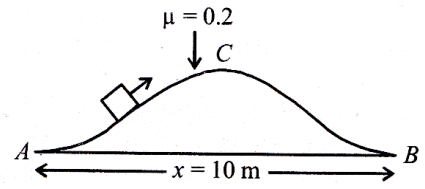
   (d) the motion is retarded and finally the particle stops

3. Two identical particles are projected horizontally in opposite directions with a speed of $5\,ms^{-1}$ each from the top of a tall tower as shown. Assuming = $10ms^{-2}$, the distance between them at the moment when their velocity vectors become mutually perpendicular is



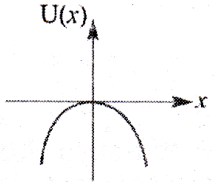
   (a) 2.5 m (b) 5m (c) 10m (d) 20m

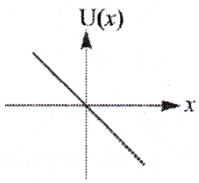
4. A block of mass 1 kg is pulled along the curve path ACB by a tangential force as shown in figure. The work done by the frictional force when the block moves from A to B is

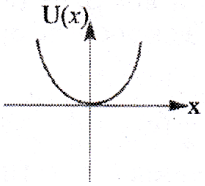


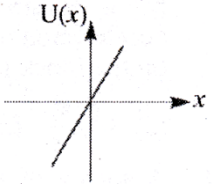
   (a) 5 J (b) 10 J (c) 20 J (d) None of these

5. A particle is placed at the origin and a force F = kx is acting on it (where k is positive constant). If U(0) = 0, the graph of U(x) versus x will be (where U is the potential energy function) :

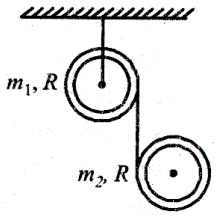
(a) 

(b) 

(c) 

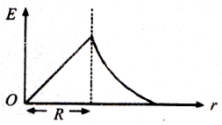
(d) 

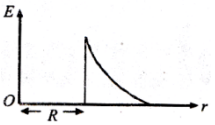
6. Linear acceleration of cylinder of mass m_2 is a_2. Then angular acceleration α_2 is (given that there is no slipping).

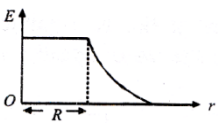


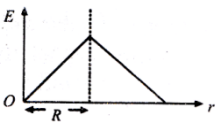
(a) $\frac{a_2}{R}$ (b) $\frac{(a_2+g)}{R}$ (c) $\frac{2(a_2+g)}{R}$ (d) None of these

7. Which one of the following plots represents the variation of gravitational field on a particle with distance r due to a thin spherical shell of radius R? (r is measured from the centre of the spherical shell)

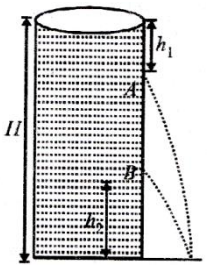
(a) 

(b) 

(c) 

(d) 

8. In a cylindrical water tank, there are two small holes A and B on the wall at a depth of h_1, from the surface of water and at a height of h_2 from the bottom of water tank. Surface of water is at height of h_2 from the bottom of water tank. Surface of water is at height H from the bottom of water tank. Water coming out from both holes strikes the ground at the same point S. Find the ratio of h_1 and h_2

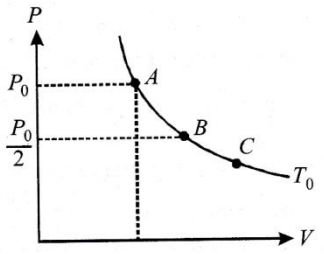


(a) Depends on H (b) 1 : 1 (c) 2 : 2 (d) 1 : 2

9. The two ends of a metal rod are maintained at temperatures 100^oC and 110^oC. The rate of heat flow in the rod is found to be 4.0 J/s. If the ends are maintained at temperature 200^oC and 210^oC, the rate of heat flow will be

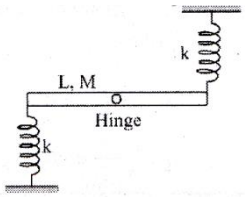
(a) 16.8 J/s (b) 8.0 J/s (c) 4.0 J/s (d) 44.0 J/s

10. The state of an ideal gas is changed through an isothermal process at temperature T_0 as shown in figure. The work done by gas in going from state B to C is double the work done by gas in going from state A to B. If the pressure of the gas in state C is



(a) $P_0/2$ (b) $P_0/4$ (c) $P_0/6$ (d) $P_0/8$

11. A rod of mass M and length L is hinged at its centre of mass so that it can rotate in a vertical plane. Two springs each of stiffness k are connected at its ends, as shown in the figure. The time period of SHM is

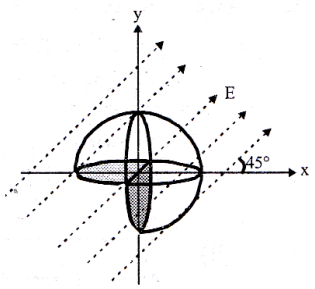


(a) $2\pi\sqrt{\frac{M}{6k}}$ (b) $2\pi\sqrt{\frac{M}{3k}}$ (c) $2\pi\sqrt{\frac{ML}{k}}$ (d) $\pi\sqrt{\frac{M}{6k}}$

12. A stretched wire 60 cm long is vibrating with its fundamental frequency of 256 Hz. If the length of the wire is decreased to 15 cm and the tension remains the same. Then the fundamental frequency of the vibration of the wire will be

(a) 1024 (b) 572 (c) 256 (d) 64

13. One-fourth of a sphere of radius R is removed as shown in figure. An electric field E exists parallel to the xy plane. Find the flux through the remaining curved part.

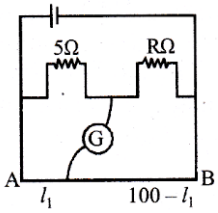


(a) xR^2E (b) $\sqrt{2}\pi R^2E$ (c) $\pi R^2E/\sqrt{2}$ (d) None of these

14. Two identical thin rings each of radius R meters are coaxially placed at a distance R meters apart. If Q_1 coulomb and Q_2 coulomb are respectively the charges uniformly spread on the two rings, the work done in moving a charge q from the centre of one ring to that of other is

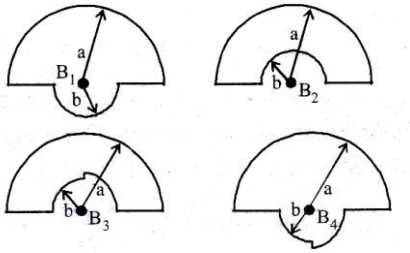
(a) zero (b) $\frac{q(Q_1-Q_2)(\sqrt{2}-1)}{\sqrt{2}.4\pi\varepsilon_0 R}$ (c) $\frac{q\sqrt{2}(Q_1+Q_2)}{4\pi\varepsilon_0 R}$ (d) $\frac{q(Q_1+Q_2)(\sqrt{2}+1)}{\sqrt{2}.4\pi\varepsilon_0 R}$

15. The resistances in the two arms of the meter bridge are 5Ω and RΩ, respectively. When the resistance R is shunted with an equal resistance, the new balance point is at $1.6l_1$. The resistance 'R' is :



(a) 10Ω (b) 15Ω (c) 20Ω (d) 25Ω

16. In the loops shown in figure. All curved sections are either semi-circles or quarter circles. All the loops carry same current. The magnetic fields at the centres have magnitudes B_1, B_2, B_3 and B_4. Then, which is correct?

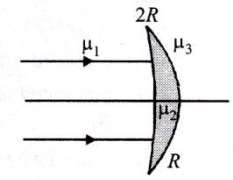


(a) $B_1 > B_2 > B_3 > B_4$ (b) $B_3 > B_4 > B_1 > B_1$ (c) $B_4 > B_1 > B_2 > B_3$ (d) $B_1 > B_4 > B_3 > B_2$

17. The magnetic susceptibility of a paramagnetic substances at -73^oC is 0.0060, then its value at -173^oC will be

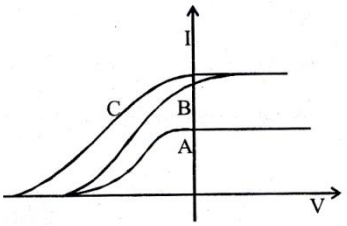
(a) 0.0030 (b) 0.0120 (c) 0.0180 (d) 0.0045

18. Figure shows a concavo-convex lens μ_2. What is the condition on the reflective indices so that the lens is diverging?



(a) $2\mu_3 < \mu_1 + \mu_2$ (b) $2\mu_3 > \mu_1 + \mu_2$ (c) $\mu_3 > 2(\mu_1 - \mu_2)$ (d) None of these

19. In a photoelectric experiment, anode potential is plotted against plate current in figure, then



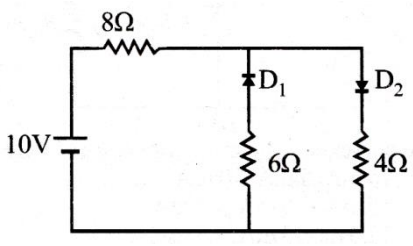
(a) A and B will have different intensities while B and C will have different frequencies

(b) B and C will have different intensities while A and C will have different frequencies

(c) A and B will have different intensities while A and C will have equal frequencies

(d) B and C will have equal intensities while A and B will have same frequencies

20. The circuit has two oppositively connected ideal diodes in parallel. What is the current flowing in the circuit?



(a) 0.4A (b) 0.8A (c) 0.6A (d) 0.5A

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. Two moles of a monoatomic gas are mixed with 'n' moles of a polyatomic gas. If mixture behaves like diatomic gas, then find the value of n.

22. A uniform magnetic field exists in a region is given by $\vec{B}=3\hat{i}+4\hat{j}+5\hat{k}$. A rod of length 5 m is placed along y-axis is moved along x-axis with constant speed 1 m/s. The induced emf in the rod will be

23. In an AC circuit a resistance R is connected in series with an inductance L. If the phase difference between the voltage and current is $\frac{\pi}{4}$, the value of inductive reactance is xR. Find the value of x.

24. Two lenses of focal lengths $f_1=10$ cm and $f_2=20\,\text{cm}$ are kept as shown. The power of combination will be

25. Activity of a substance changes from $700s^{-1}$ to $500s^{-1}$ in 30 minute. Find its half-life in minutes.

Chemistry

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. Saponification of ethyl acetate ($CH_3COOC_2H_5$) by NaOH is studied by titration of the reaction mixture have 1 : 1 molar ratio of the reactants. If 10 mL of 1 N HCl is required by 5 mL of the solution at the start and 8 M of 1N HCl is required by another 5 mL after 10 minutes.

(a) $k = \frac{2.303}{10}\log\frac{10}{8}$ (b) $k = \frac{2.303}{10}\log\frac{10}{2}$ (c) $k = \frac{1}{10}\left[\frac{1}{8}-\frac{1}{10}\right]$ (d) $k = \frac{1}{10}\left[\frac{1}{2}-\frac{1}{10}\right]$

2. There is no S-S bond in

(a) $S_2O_4^{2-}$ (b) $S_2O_5^{2-}$ (c) $S_2O_3^{2-}$ (d) $S_2O_7^{2-}$

3. The maximum oxidation state shown by V(Z = 23), Cr(Z = 24), Co(Z = 27), Sc(Z = 21) are respectively:

(a) +5, +6, +3, +3 (b) +3, +4, +5, +2 (c) +5, +3, +2, +1 (d) +4, in each case

4. The correct name of $[Pt(NH_3)_4Cl_2][PtCl_4]$ is

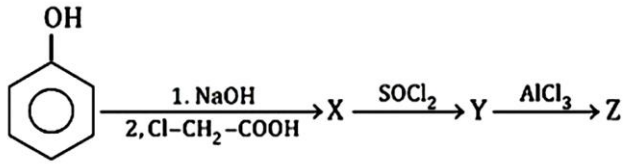
(a) tetraammine dichloro platinum (IV) tetrachloroplatinate (II)

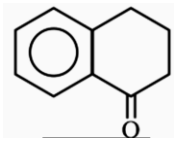
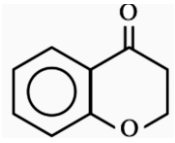
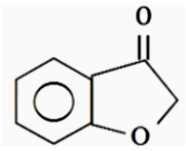
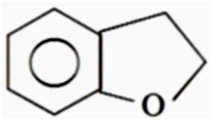
(b) dichlorido tetra ammine platinium (IV) tetrachloridoplatinate (II)

(c) tetrachlorido platinum (II) tetraammineplatinate (IV)

(d) tetrachlorido platinum (II) dichloridotetraammineplatinate (IV)

5. Identify 'Z' in the given sequence of reaction.



(a)  (b)  (c)  (d) 

6. When BrO_3^- ion reacts with Br^- ion in acid medium Br_2 is liberated. The equivalent mass of Br_2 in the reaction is:

(a) $\frac{5M}{3}$ (b) $\frac{3M}{5}$ (c) $\frac{4M}{6}$ (d) $\frac{5M}{8}$

7. How fast is an electron moving if it has a wavelength equal to the distance it travels in 1 second?

(a) $\sqrt{\frac{m}{h}}$ (b) $\sqrt{\frac{h}{2KE}}$ (c) $\sqrt{\frac{h}{m}}$ (d) $\sqrt{\frac{h}{p}}$

8. Identify the correct statement regarding a spontaneous process

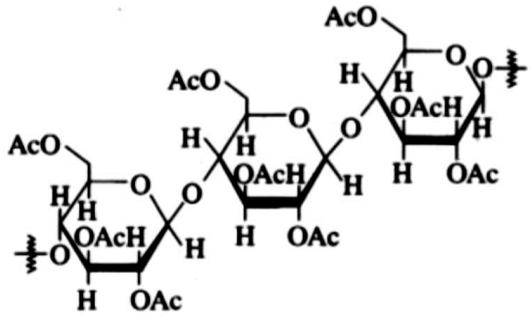
(a) For a spontaneous process in an isolated system, the change in entropy is positive

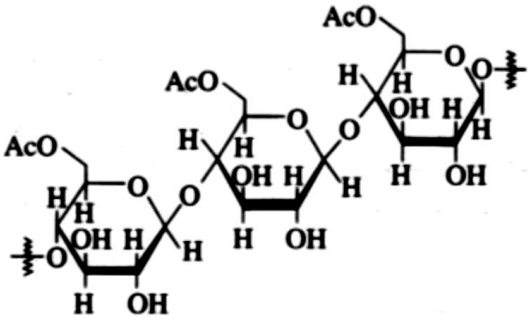
(b) Endothermic processes are never spontaneous

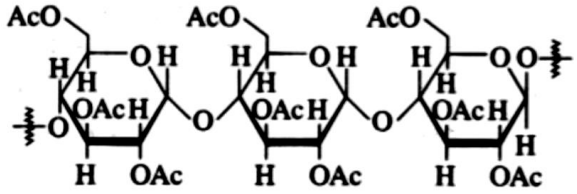
(c) Exothermic processes are always spontaneous

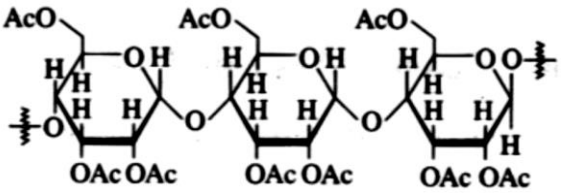
(d) Lowering of energy in the reaction process is the only criterion for spontaneity

9. Cellulose upon acetylation with excess acetic anhydride/H_2SO_4 (catalytic) give cellulose triacetate whose structure is

(a) 

(b) 

(c) 

(d) 

10. 1.2 kg ethylene glycol $\begin{pmatrix} CH_2OH \\ | \\ CH_2OH \end{pmatrix}$ was added in a car radiator containing 9 litres water. The freezing of water was just prevented when car was running in the Himalayan valley at temperature –4°C. Sudden thunderstorm in the valley lowered the temperature to –6°C. Calculate the amount of ice separated.

(a) 1 kg (b) 2 kg (c) 3 kg (d) 4 kg

11. Resistance of 0.2 M solution of an electrolyte is 50 Ω. The specific conductance of the solution is 1.3 S m^{-1}. If resistance of the 0.4 M solution of the same electrolyte is 260 Ω, its molar conductivity is

(a) 6250 S m^2 mol^{-1} (b) 6.25×10^{-4} S m^2 mol^{-1}

(c) 625×10^{-4} S m^2 mol^{-1} (d) 62.5 S m^2 mol^{-1}

12. To an equilibrium mixture of

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$$

some helium, an inert gas, is added at constant volume. The addition of helium causes the total pressure to double. Which of the following is true?

(a) The concentration of three gases is unchanged

(b) The concentration of Sulphur trioxide increases

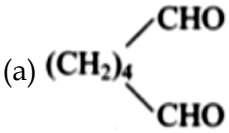
(c) The number of moles of Sulphur trioxide increases

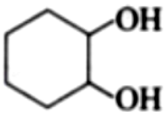
(d) The concentration of Sulphur dioxide increases

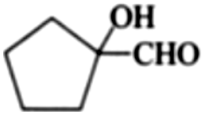
13. The first ionization enthalpies of Na, Mg, Al and Si are in the order:

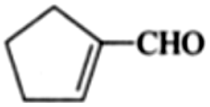
(a) Na < Mg > Al < Si (b) Na > Mg > Al > Si (c) Na < Mg < Al < Si (d) Na > Mg > Al < Si

14. 

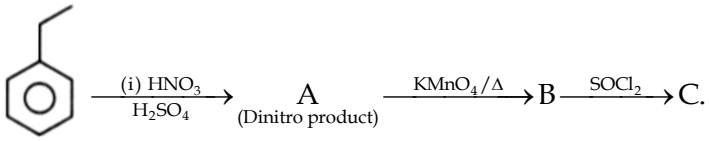
(a) 

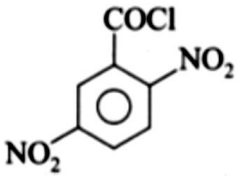
(b) 

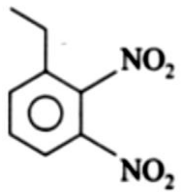
(c) 

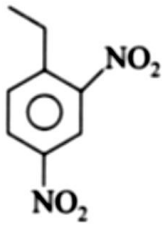
(d) 

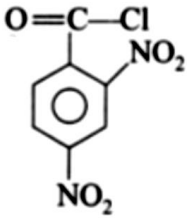
15. Product C of the given reaction is

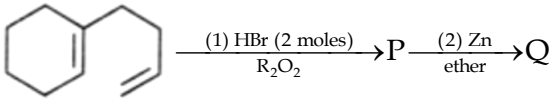


(a) 

(b) 

(c) 

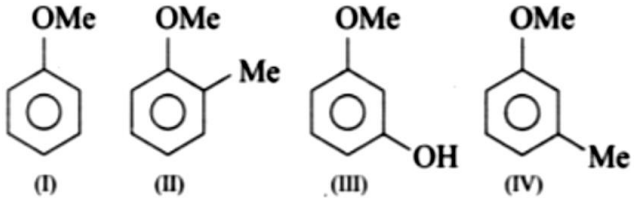
(d) 

16. 

Give the molecular weight of Q:

(a) 135 (b) 140 (c) 138 (d) 133

17. Arrange the compounds in decreasing order of reactivity towards bromine:



(a) III > II > IV > I (b) II > IV > I > III (c) III > IV > II > I (d) IV > II > I > III

18. Be and Al exhibit many properties which are similar, but the two elements differ in

(a) Exhibiting amphoteric nature in their oxides (b) Forming polymeric hydrides

(c) Exhibiting maximum covalency in compounds (d) Forming covalent halides

19. H_3BO_3 ionises in water as:

(a) $H_3BO_3(aq) \rightleftharpoons H^+(aq) + H_2BO_3^-(aq)$ (b) $H_3BO_3(aq) \rightleftharpoons 2H^+(aq) + HBO_3^{2-}(aq)$

(c) $H_3BO_3(aq) \rightleftharpoons 3H^+(aq) + BO_3^{3-}(aq)$ (d) None of these

20. For the following acids

$\underset{(I)}{(CH_3)_3CCH_2CO_2H}$ $\underset{(II)}{(CH_3)_3\overset{\oplus}{N}CH_2CO_2H}$ $\underset{(III)}{CH_3COOH}$

pK_a value will be in order

(a) I > II > III (b) I < II < III (c) I > III > II (d) II > III > I

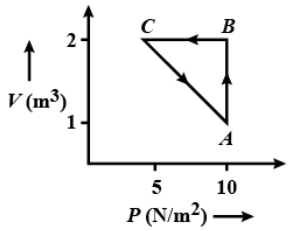
(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. 100 mL of 0.15 M solution of $CoCl_3.xNH_3$ was treated with excess of $AgNO_3$ solution and 0.03 moles of AgCl was obtained, then find out value of x.

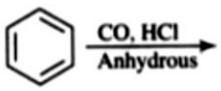
22. In 1 L saturated solution of AgCl [$K_{sp}(AgCl) = 1.6 \times 10^{-10}$], 0.1 mol of CuCl [$K_{sp}(CuCl) = 1.0 \times 10^{-6}$] is added. The resultant concentration of Ag^+ in the solution is 1.6×10^{-x}. The value of "x" is

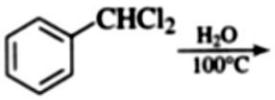
23. An ideal gas is taken through the cycle A → B → C → A as shown in the figure. If net heat supplied to the gas in the cycle is 5J, the work done on the gas is the process C → A is _____ J.

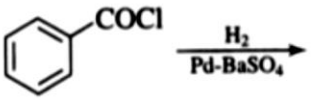


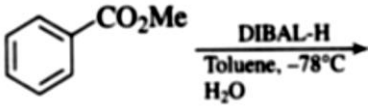
24. An organic compound undergoes first-order decomposition. The time take for its decomposition to 1/8 and 1/10 of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of $\left[\frac{t_{1/8}}{t_{1/10}}\right] \times 10$? ($\log_{10}2 = 0.3$)

25. Among the following the number of reaction(s) that produce(s) benzaldehyde is

I. 

II. 

III. 

IV. 

Mathematics

(Single Correct Choice Type)

This Section contains **20 multiple choice questions.** Each question has four choices (a), (b), (c) and (d) out of which **ONLY ONE** is correct.

1. The equation of circle which touches both the axes and the straight line $4x+3y=6$ in the first quadrant and lies below it.

   (a) $4x^2+4y^2-4x-4y+1=0$ (b) $(x-3)^2+(y-3)^2=9$

   (c) $(x-1)^2+(y-1)^2=1$ (d) $(x-4)^2=(y-4)^2=16$

2. The four straight lines given by the equations $12x^2+7xy-12y^2=0$ and $12x^2+7xy-12y^2-x+7y-1=0$ lie along the sides of a

   (a) square (b) rhombus (c) rectangle (d) parallelogram

3. Suppose $a,b\in R$. If the equation $x^2-(2a+b)x+(2a^2+b^2-b+1/2)=0$ has two real roots, then

   (a) $a=\frac{1}{2},b=-1$ (b) $a=-\frac{1}{2},b=1$ (c) $a=2,b=1$ (d) $a=-2,b=-1$

4. If $0<a<b<c<d$, then the quadratic equation $ax^2+\{1-a(b+c)\}x+abc-d=0$ has

   (a) real and distinct roots out of which one lies be tween c and d.

   (b) real and distinct roots out of which one lies be , tween a and b

   (c) real and distinct roots out of which one lies be, tween b and c

   (d) non-real roots.

5. Let $f(x)=\frac{x^2-2x+3}{x^2-2x-8},x\in R-\{-2,4\}$ The range of f is

   (a) $\left(\frac{-2}{9},1\right]$ (b) $R-\left(\frac{-2}{9},1\right)$ (c) $\left(-\infty,\frac{-2}{9}\right]$ (d) $R-\left(\frac{-2}{9},1\right]$

6. If $\sum_{r=1}^{n}t_r=\frac{1}{12}n(n+1)(n+2)$, the value $\sum_{r=1}^{n}\frac{1}{t_r}$ is

   (a) $\frac{2n}{n+1}$ (b) $\frac{n-1}{(n+1)!}$ (c) $\frac{4n}{n+1}$ (d) $\frac{3n}{n+2}$

7. Let $I_n=\int_0^{\pi/4}\tan^n x\,dx$. Then $I_2+I_4+I_3+I_5, I_4+I_6, I_5+I_7,.....$ are in

   (a) A.P. (b) G.P. (c) H.P. (d) None of these

8. Sum to n terms of the series $\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\frac{7}{4.5.6}+.....$ is

   (a) $\frac{n(n+1)}{2(n+2)(n+3)}$ (b) $\frac{n(3n+1)}{4(n+1)(n+2)}$ (c) $\frac{1}{6}-\frac{5}{(n+1)(n+4)}$ (d) none of these

9. If $f(x)=\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ then $\lim_{x\to\infty}f(x)$ is

   (a) e^{-4} (b) e^3 (c) e^2 (d) e^4

10. If $f(x)=\begin{cases}x^2+Ax+5 & x\in Q\\ 1+x & x\in R\sim Q\end{cases}$ is continuous at exactly two points, then he possible values of A are in

    (a) $(1,\infty)$ (b) $(-3,\infty)$ (c) $[5,\infty)\cup(-\infty,-3]$ (d) $(-\infty,\infty)$

11. Let $f(x)=\dfrac{\sin(\pi\cos^2 x)}{x^2}$, $x\neq 0$. The value of $f(0)$ so that f is a continuous function is

(a) $-\pi$ (b) π (c) $\pi/2$ (d) 1

12. Let $h(x)=\min\{x, x^2\}$ for $x\in R$. Then which of the following is not correct

(a) h is continuous for all x (b) h is differentiable for all x

(c) h′(x) =1 for all x > 1

(d) h is not a differentiable function at atleast two points

13. If $f(x)=\sqrt{x=2\sqrt{2x-4}}+\sqrt{x-2\sqrt{2x-4}}$ then

(a) f is differentiable at all points of its domain except x = 4

(b) f is differentiable on $(2,\infty)$

(c) f is differentiable on $(-\infty,\infty)$

(d) $f'(x)=0$ for all $x\in[2,0)$

14. If $y=(x^2+1)^{\sin x}$, then $y'(0)$ is equal to

(a) 1/2 (b) e^2 (c) 0 (d) 3/2

15. $\dfrac{d^2y}{dy^2}$ equals

(a) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$ (b) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$ (c) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ (d) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$

16. Let $P=\{\theta:\sin\theta-\cos\theta=\sqrt{2}\cos\theta\}$ and Q = $\{\theta:\sin\theta+\cos\theta=\sqrt{2}\sin\theta\}$ be two sets, then

(a) $P\subset Q$ and $Q\sim P\neq\phi$ (b) $Q\not\subset P$

(c) $P\not\subset Q$ (b) $P=Q$

17. Let $R=\{(x,y):x,y\in R, x^2+y^2\leq 25\}$ $R'=\left\{(x,y):x,y\in R, y\geq\dfrac{4}{9}x^2\right\}$ then

(a) $domR\cap R'=[-4,4]$ (b) range $R\cap R'=[0,4]$

(c) range $R\cap R'=[0,5]$ (d) $R\cap R'$ defines a function.

18. If $f(x)^2 f\left(\dfrac{1-x}{1+x}\right)=x^3, x\neq -1,1$ and $f(x)\neq 0$, then $\{f(-2)\}$ (the fractional part of $f(-2)$) is equal to

(a) 2/3 (b) 1/3 (c) 1/2 (d) 0

19. The domain of the function $f(x)=\log_2\left(-\log_{1/2}\left(1+\dfrac{1}{\sqrt[4]{x}}\right)-1\right)$ is

(a) $0<x<1$ (b) $0<x\leq 1$ (c) $x\geq 1$ (d) $x>1$

20. Let $A=\{x\in R:[x+3]+[x+4]\leq 3\}$ and $B=\left\{x\in R:3^x\left(\sum_{r=1}^{\infty}\dfrac{3}{10^r}\right)^{x-3}<3^{-3x}\right\}$ then

(a) $A=B$ (b) $A\underset{\neq}{\subset}B$ (c) $B\underset{\neq}{\subset}A$ (d) $A\cap B=\phi$

(Integer Type Questions)

This Section contains **05 Questions.** The answer to each question is a single digit integer ranging from 0 to 9. The correct digit below the question number in the ORS is to be bubbled.

21. The line $3x+6y=k$ intersects the curve $2x^2+2xy+3y^2=1$ at points A & B. The circle on AB as diameter passes through the origin. Them the value of k^2 is

22. Let $A(-4,0)$ and $B(4,0)$. It the number of points on the circle $x^2+y^2=16$ such that area of triangle whose vertices are A, B and C is a positive integer N, then N is

23. The sum of the square of the length of the chord intercepted by the line $x+y=n$, $n\in N$ on the circle $x^2+y^2=4$ is

24. Consider the family of lines $(5x+3y-2)+\lambda(3x-y-4)=0$ and $(x-y+1)+\mu(2x-y-2)=0$ Equation of straight line that belong to both families is $ax+by-7=0$ then $a+b$ is -(b)

25. If $2x^2+3xy+by^2-11x+13y+c=0$ represent pair of perpendicular straight lines then $|b+2c|$ is .

Answer – key

Physics

1.	C
2.	B
3.	C
4.	C
5.	D
6.	B
7.	B
8.	D
9.	B
10.	B
11.	A
12.	B
13.	B
14.	C
15.	D
16.	A
17.	D
18.	C
19.	C
20.	A
21.	5
22.	9
23.	0
24.	3
25.	1

Chemistry

1.	C
2.	D
3.	A
4.	A
5.	C
6.	B
7.	C
8.	A
9.	A
10.	C
11.	B
12.	A
13.	A
14.	D
15.	D
16.	C
17.	C
18.	C
19.	D
20.	C
21.	5
22.	7
23.	-5
24.	9
25.	4

Math

1.	a
2.	a
3.	a
4.	a
5.	d
6.	c
7.	c
8.	b
9.	d
10.	c
11.	b
12.	b
13.	a
14.	c
15.	a
16.	d
17.	c
18.	a
19.	a
20.	a

Integer

21.	9
22.	62
23.	22
24.	3
25.	6

Solution

Physics

1. (b)

Given $R = \frac{\rho \ell}{\pi r^2}$, then

$$\frac{\Delta R}{R} \times 100$$
$$= \frac{\Delta \rho}{\rho} \times 100 + \frac{\Delta \ell}{\ell} \times 100 + 2\frac{\Delta r}{r} \times 100$$
$$= 1\% + 2\% + 2 \times 3\% = 9\%$$

2. (d)

From displacement-time graph, it is clear that in equal intervals of time displacements are not equal infact, decreases and after 40s displacement constant i.e. the particle stops.

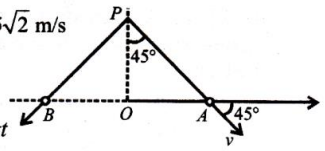
3. (b)

At a time t when velocity vector become mutually perpendicular

$v \cos 45° = 5$ horizontal component

$$v = \frac{5}{\cos 45°} = 5\sqrt{2} \text{ m/s}$$

Vertically,



$v \sin 45° = gt$

$$\Rightarrow t = \frac{v \sin 45°}{g} = \frac{5\sqrt{2} \times 1/\sqrt{2}}{10} = \frac{5}{10} = \frac{1}{2}$$

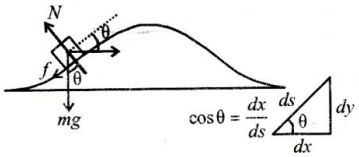
so, $OA = OB = v \cos 45° \times t = 5 \times \frac{1}{2} = 2.5$

$\Rightarrow \quad AB = 2.5 \times 2 = 5$ m

4. (c)

Work done by friction

$$= \int \vec{F} \cdot \vec{ds} = \int_0^x \mu mg \cos\theta \frac{dx}{\cos\theta} = \mu mg\, x = 20 \text{ J}$$



5. (a)

$$U = -\int_0^x F dx = -\int_0^x kx dx = -\frac{1}{2}kx^2.$$

It is correctly drawn in (a)

6. (c)

$$m_2 g - T = m_2 a_2 \quad \text{...(i)}$$
$$TR = \frac{m_2 R^2}{2}\alpha_2 \quad \text{...(ii)}$$
$$\alpha_1 R = a_2 - \alpha_2 R \quad \text{...(iii)}$$
$$TR = \left(\frac{m_1 R^2}{2}\right)\alpha_1 \quad \text{...(iv)}$$
$$\alpha_2 = \frac{2T}{m_2 R} = \frac{2}{m_2 R}.(m_2 a_2 + m_2 g)$$
$$= \frac{2(a_2 + g)}{R}$$

7. (b)

The Gravitational field due to a thin spherical shell of radius R at distance r.

$$E = \frac{GM}{r^2} \text{ (If } r > R\text{)}$$

For $r = R$ i.e. on the surface of the shell

$$E = \frac{GM}{R^2}$$

For $r < R$ i.e. inside the shell

$E = 0$

8. (b)

Range is same for both holes

$\therefore \quad 2\sqrt{(H - h_1)h_1} = 2\sqrt{(H - h_2)h_2}$

Squaring both sides,

$4(H - h_1)\, h_1 = 4(H - h_2)\, h_2$

$Hh_1 - h_1^2 = Hh_2 - h_2^2$

On solving we get,

$H = h_1 + h_2$ (not possible) and $h_1 - h_2 = 0$

Hence, the ratio of $\frac{h_1}{h_2} = 1:1$.

9. (c)

As the temperature difference $\Delta T = 10°C$ as well as the thermal resistance is same for both the cases, so thermal current or rate of heat flow will also be same for both the cases.

10. (d)

Work done by gas in going isothermally from state A to B is

$$\Delta W_{AB} = nRT \ln \frac{P_A}{P_B} = nRT \ln 2 \quad \text{...(i)}$$

Work done by gas in going isothermally from state B to C is

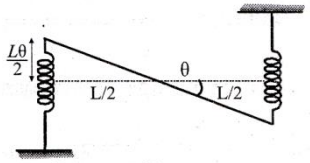
$$\Delta W_{BC} = nRT \ln \frac{P_B}{P_C} = nRT \frac{P_0}{2P_C} \quad \text{...(ii)}$$

It is given that $\Delta W_{BC} = 2\Delta W_{AB}$

$$\ln \frac{P_0}{2P_C} = \ln(2)^2 \qquad \therefore \quad P_C = \frac{P_0}{8}$$

11. (a)

The restoring torque (for small θ)



$$\tau_{rest} = -\left[\frac{kL\theta}{2}\times\frac{L}{2}\right]\times 2 = \frac{kL^2}{2}(-\theta)$$

$$\therefore\ \alpha = \frac{\tau_{rest}}{I} = \frac{kL^2/2}{ML^2/12}(-\theta) = \frac{6k}{M}(-\theta)$$

$$\therefore\ T = 2\pi\sqrt{\frac{M}{6k}}.$$

12. (a)

$L_0 = 60\text{ cm}$ $\qquad \nu_0 = 256\text{ Hz}.$

$$\nu = \frac{1}{2L}\sqrt{\frac{T}{m}} \qquad \therefore\ \nu \propto \frac{1}{L}$$

$$\frac{\nu_1}{\nu_0} = \frac{L_0}{L_1}$$

$$\Rightarrow\ \nu_1 = \nu_0\frac{L_0}{L_1} = 256\times\frac{60}{15} = 1024\text{ Hz}.$$

13. (c)

$$\phi_{plain} + \phi_{curve} = 0 \text{ or } \phi_{plain} = -\phi_{curve}$$

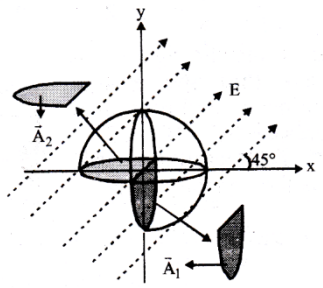
$$\vec{A}_1 = -\frac{\pi R^2}{2}\hat{i}, \vec{A}_2 = -\frac{\pi R^2}{2}\hat{j}$$

$$\vec{E} = E\cos 45°\hat{i} + E\sin 45°\ \hat{j}$$

$$= \frac{E}{\sqrt{2}}\hat{i} + \frac{E}{\sqrt{2}}\hat{j} \text{ and}$$

$$\phi = \vec{E}\cdot(\vec{A}_1 + \vec{A}_2)$$

$$= \frac{-E}{\sqrt{2}}\frac{\pi R^2}{2} - \frac{E}{\sqrt{2}}\frac{\pi R^2}{2} = \frac{-\pi R^2 E}{\sqrt{2}}$$



This is the flux entering. So flux is $\frac{\pi R^2 E}{\sqrt{2}}$

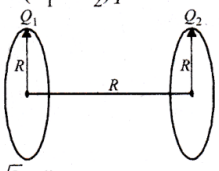
14. (b)

Work done $W_{21} = (V_1 - V_2)q$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1}{R} + \frac{Q_2}{\sqrt{2}R}\right]$$

and

$$V_2 = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_2}{R} + \frac{Q_1}{\sqrt{2}R}\right]$$



Thus, $W_{21} = \frac{q(Q_1 - Q_2)(\sqrt{2}-1)}{\sqrt{2}.4\pi\epsilon_0 R}$.

15. (b)

This is a balanced wheatstone bridge condition,

$$\frac{5}{R} = \frac{\ell_1}{100-\ell_1} \text{ and } \frac{5}{R/2} = \frac{1.6\ell_1}{100-1.6\ell_1}$$

$$\Rightarrow\quad R = 15\,\Omega$$

16. (c)

17. (b)

As magnetic susceptibility $\chi_m \propto \frac{1}{T}$, therefore

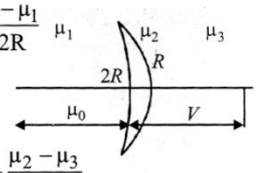
$$\frac{\chi_2}{\chi_1} = \frac{T_1}{T_2} \Rightarrow \frac{\chi_2}{0.0060} = \frac{273-73}{273-173} = \frac{200}{100} = 2$$

$$\chi_2 = 2\times 0.0060 = 0.0120$$

18. (b)

$$\frac{\mu_2}{V} - \frac{\mu_1}{-u_0} = \frac{\mu_2-\mu_1}{-2R}$$

$$\frac{\mu_3}{V_f} - \frac{\mu_2}{V} = \frac{\mu_3-\mu_2}{-R}$$



$$\frac{\mu_3}{V_f} + \frac{\mu_1}{u_0} = \frac{\mu_1-\mu_2}{2R} + \frac{\mu_2-\mu_3}{R}$$

$$\therefore\quad \frac{\mu_1-\mu_2}{2} < \mu_3-\mu_2 \Rightarrow \mu_1-\mu_2 < 2\mu_3 - 2\mu_2$$

$$\Rightarrow\quad \mu_1+\mu_2 < 2\mu_3$$

19. (d)

From the graph it is clear that *A* and *B* have the same stopping potential and therefore, the same frequency. Also, *B* and *C* have the same intensity.

20. (b)

D_2 is forward biased whereas D_1 is reversed biased.

So effective resistance of the circuit

$$R = 8+4 = 12\Omega \qquad \therefore i = \frac{10}{12} = 0.8\text{ A}$$

21. (4)

Using, $\quad f = \frac{n_1 f_1 + n_2 f_2}{n_1 + n_2}$

or $\quad 5 = \frac{2\times 3 + n\times 6}{2+n}$

$\therefore\quad n = 4.$

22. (25)

$$e = (\vec{v}\times\vec{B}).\vec{\ell} = [\hat{i}\times(3\hat{i}+4\hat{j}+5\hat{k})].5\hat{j}$$

$= 25$ V.

23. (1)

$$\tan\frac{\pi}{4}=\frac{X_L}{R} \quad \Rightarrow \quad X_L=R.$$

24. (10)

$$\frac{1}{f_e}=\frac{2}{f_1}+\frac{2}{f_2}+\frac{1}{f_m}$$

or
$$P=\frac{2}{0.10}+\frac{2}{-0,20}+\frac{1}{\infty}$$
$$=10\ \text{D}.$$

25. (62)

$$\ell n\left[\frac{A_0}{A_t}\right]=\lambda t$$

$$\Rightarrow \quad \ell n2=\lambda t_{1/2} \qquad ...(i)$$

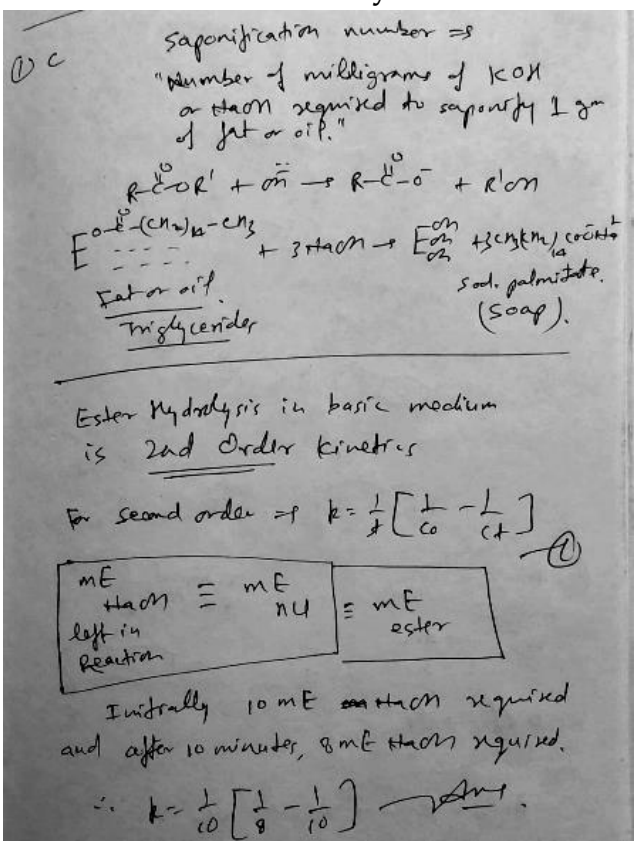
$$\Rightarrow \quad \ell n\left[\frac{700}{500}\right]=\lambda(30\ \text{min}) \qquad ...(ii)$$

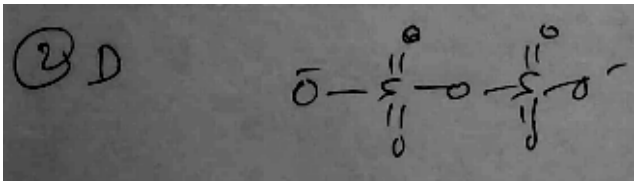
(i)/(ii)

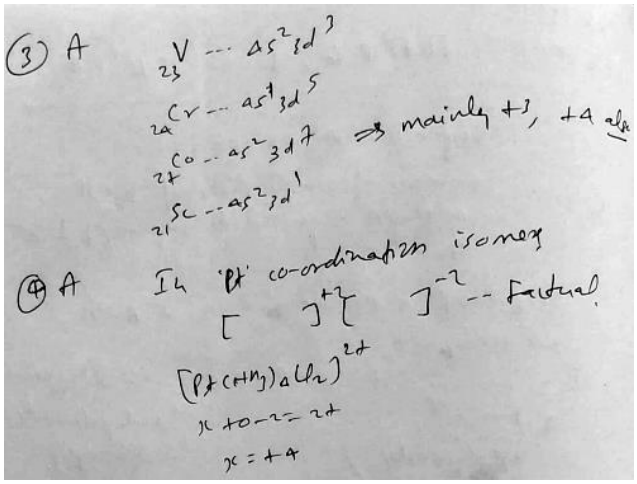
$$\Rightarrow \quad \frac{\ell n2}{\ell n(7/5)}=\frac{t_{1/2}}{(30\,\text{min})}$$

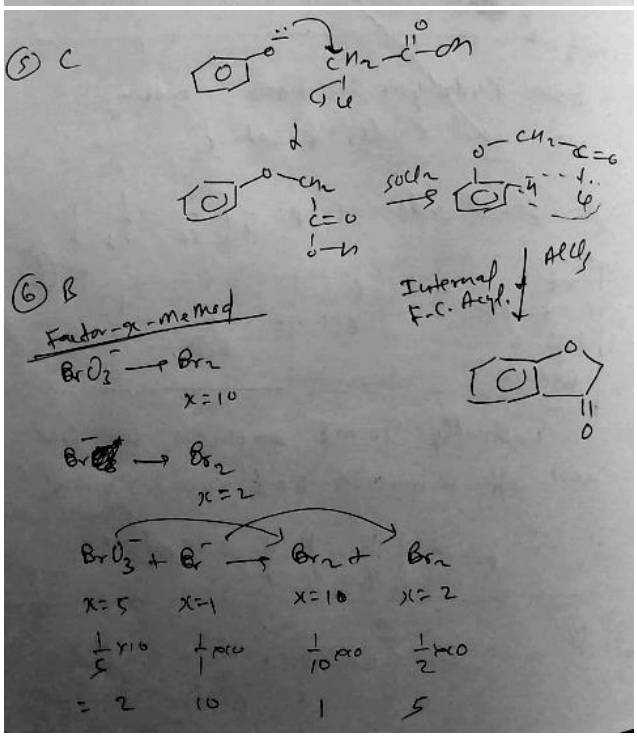
$$\Rightarrow \quad (2.06004)\,30=t_{1/2}=61.8\ \text{min}.$$

Chemistry









$2BrO_3^- + 10Br^- \rightarrow 6Br_2$; $gE = n \times x$

exs = 10gE 10×1 = 10gE 10gE $\therefore n = \frac{gE}{x}$

mole = 1

$x = \frac{10}{6} = \frac{5}{3}$ & $x = \frac{gE}{n}$

Hence, $E = \frac{F}{x} = \frac{F}{5/3} = \frac{3F}{5}$ — Ans.

General Method

$+2BrO_3^- \rightarrow Br_2 + 6H_2O$ — Reduction

$12H^+ + 10e^-$

$(2Br^- \rightarrow Br_2 + 2e^-) \times 5$ — Oxid.

$2BrO_3^- + 12H^+ + 10Br^- \rightarrow 6Br_2 + 6H_2O$

electrons involved = 10

$E = \frac{M}{x}$; $x = \frac{e^-}{\text{coefficient}}$

$x_{Br_2} = \frac{10}{6} = \frac{5}{3} \Rightarrow \therefore E = \frac{M}{5/3} = \frac{3M}{5}$ Ans

(7) C $\lambda = \frac{h}{mv}$ $v = \lambda$ ---given

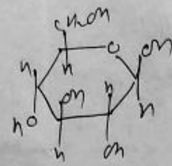
$\therefore v^2 = \frac{h}{m}$

(8) A $\Delta S_{Total} = +ve$ for spont.

$\Delta S_{Total} = \Delta S_{system} + \Delta S_{surr}$

$= \Delta S_{system} + 0$

(9) A Cellulose is polymer of β-D-glucose



This is β-D-glucose.

(10) C $\Delta T_f = i K_f m$ $(CH_2OH)_2$ Mol. mass = 62

$4 = 1 \times K_f \times \frac{1200/62}{9}$ — (I) $m = \frac{n}{w_{H(kg)}}$

$6 = 1 \times K_f \times m'$ — (II)

$\frac{2}{3} = \frac{1200}{62 \times 9 \times m'} \Rightarrow m' = \frac{1200 \times 3}{2 \times 62 \times 9} \approx 3.3$

$m' = 3.3 = \frac{1200/62}{w_{H(kg)}} \Rightarrow w_{H(kg)} = \frac{1200}{62 \times 3.3}$

$= 5.8$ kg

$\therefore$ Separated ice $= 9 - 5.8 = 3.2$ kg Ans

(11) B $R = \frac{1}{K} G^*$ $K = G G^*$

$\therefore G^* = KR$ $G^* = KR$

$= 1.3 \times 50$ $\Lambda_m = \frac{K}{M \times 1000}$ --- Scm^{-1}

$= 65\ m^{-1}$ — (I) $S\,cm^2\,mol^{-1}$

$K_{(for\ 0.4M)} = \frac{G^*}{R} = \frac{65}{260}$ Scm^{-1}

$\therefore \Lambda_m = \frac{65/260}{0.4 \times 1000} = 6.25 \times 10^{-4}$ $S\,m^2\,mol^{-1}$ Ans.

(12) A No change when

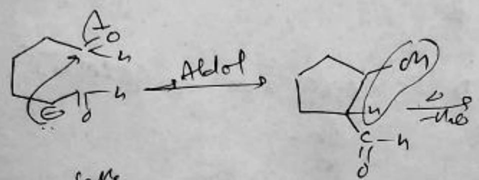
→ $\Delta n_g = 0$

→ V = const.

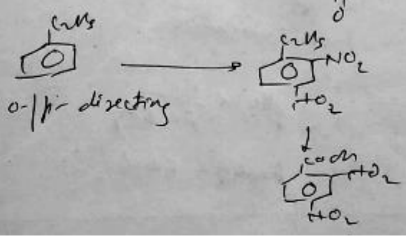
(13) A $Mg \Rightarrow 1s^2 2s^2 2p^6 3s^2$

IV more stable

(14) BD

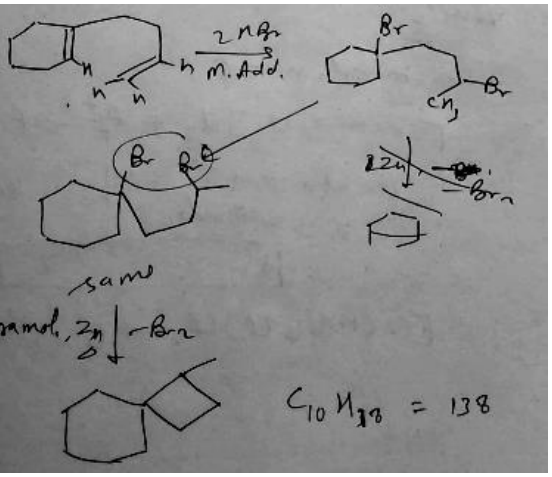


(15) D



o-/p- directing

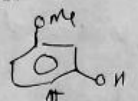
(16) C



(17) C

(i) Activating order $-OR > -CH_3$

(ii)



Reinforced by both.
Recall Korner's Rule

(18) B

(19) D

$H_3BO_3 + OH^- \rightleftharpoons [B(OH)_4]^-$

$\not\Rightarrow$
no ionisation

(20) C

$pK_a \propto \frac{1}{\text{Acidity}}$

Acidity $\propto$ EWG

(21) Ans '5'

Gen. coord. no = 6

$[Co(NH_3)_xCl_y]Cl_z + Ag^+ \rightarrow AgCl$

100 ml, 0.15 M
$\equiv$ 15 milimole

30 milli-mol.

1 : 2

$\therefore [Co(NH_3)_5Cl]Cl_2 \Rightarrow [\quad]^{2+} + 2Cl^-$

Ans (2)

simultaneous solubility

I Approx method.

$K_{sp}(CuCl) >> K_{sp}(AgCl)$,

Hence $[Cl^-]$ comes mainly from CuCl

$K_{sp}(CuCl) = 10^{-6} = [Cu^+][Cl^-] = x^2$

$\therefore x = 10^{-3} = [Cl^-]$ — (1)

$\therefore [Ag^+] = \frac{K_{sp}(AgCl)}{[Cl^-]} = \frac{1.6\times10^{-10}}{10^{-3}} = 1.6\times10^{-7}$

II Exact method

		K_{sp}
$CuCl \rightleftharpoons Cu^+ + Cl^-$		
a	(a+b)	10^{-6}
$AgCl \rightarrow Ag^+ + Cl^-$		1.6×10^{-10}
b	(b+a)	

$10^{-6} = a(a+b)$
$1.6\times10^{-10} = b(a+b)$
$\Rightarrow a = \frac{10^4}{1.6}b$ — (1)

$b(a+b) = 1.6\times10^{-10}$

$b\left(\frac{10^4}{1.6}b + b\right) = 1.6\times10^{-10}$

$\approx b \times \frac{10^4 b}{1.6} = 1.6\times10^{-10}$

$b^2 = \frac{1.6\times10^{-10}\times1.6}{10^4} = 1.6\times1.6\times10^{-14}$

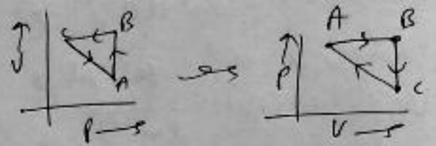
$\therefore b = 1.6\times10^{-7}$

(23) Ans. (5)

In a cyclic process, $\Delta E = 0$

$\therefore |w| = |Q| = 5$ Joule.

Heat supplied to system = work done by system



$\checkmark$ XCAT !!!!

Pl. note, ACW cycle turns CW cycle when VP graph is turned to PV graph. Clearly, net work is done "by" system.

(24) Ans.

$t_{1/8} = \frac{2.303}{k} \log \frac{a}{\frac{1}{8}a}$ —(I)

$t_{1/10} = \frac{2.303}{k} \log \frac{a}{\frac{1}{10}a}$ —(II)

$\therefore \frac{t_{1/8}}{t_{1/10}} = \frac{\log 8}{\log 10} = \frac{0.9}{1}$

$\therefore 0.9 \times 10 = 9$

(25) Ans 4

$C_6H_6 \xrightarrow[\text{dry}]{CO, HCl} C_6H_5CHO$ Gattermann Koch

$C_6H_5CHCl_2 \xrightarrow[\Delta]{H_2O} C_6H_5CH(OH)_2 \rightarrow PhCHO$

$C_6H_5COCl \xrightarrow[BaSO_4]{H_2/Pd} Ph\text{-}CHO$ Rosenmund reaction

$C_6H_5COOMe \xrightarrow{DIBAL\text{-}H} Ph\text{-}CHO$

Math

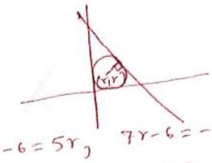
(6) Qu ① According to question equation of circle which touches both the axes and the line $4x+3y=6$

$\frac{|4r+3r-6|}{5} = r$

$|7r-6| = 5r$

$7r-6 = \pm 5r$, $7r-6=5r$, $7r-6=-5r$

$2r=6 \Rightarrow r=3$ $\quad 12r=6$, $r=\frac{1}{2}$



$(x-3)^2+(y-3)^2=9$

2Que ⇒ Given four straight lines $12x^2+7xy-12y^2=0$

ⓐ & $12x^2+7xy-12y^2-x+7y-1=0$

According to $12x^2+7xy-12y^2=0$ thats why lines are $\perp^r$ to each other.

$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$

Here $a=12$, $b=-12$, $a+b=0$

also in second eqn. $12x^2+7xy-12y^2-x+7y-1=0$

$a=12$, $b=-12$

$a+b=0$

According to condition each lines are $\perp^r$ to each other thats why quadrilateral represent a square.

Qu③ : $a, b \in R$ $\quad x^2-(2a+b)x+(2a^2+b^2-b+\frac{1}{2})=0$ has two real roots

$D \geq 0$

$(2a+b)^2 - 4\times1(2a^2+b^2-b+\frac{1}{2}) \geq 0$

$4a^2+b^2+4ab-8a^2-4b^2+4b-2 \geq 0$

$-4a^2+4ab-3b^2+4b-2 \geq 0$

$4a^2-4ab+3b^2-4b+2 \leq 0$

$(2a-b)^2+2b^2-4b+2 \leq 0$

$(2a-b)^2+2(b^2-2b+1) \leq 0$

$(2a-b)^2+2(b-1)^2 \leq 0$

$b=1$, $2a=b$, $a=\frac{1}{2}$

④ If $0<a<b<c<d$

ⓐ $f(x) = ax^2+(1-a(b+c))x+abc-d=0$

$f(a) = a^3+(1-ab-ac)a+abc-d$

$f(b) = ab^2+(1-ab-ac)b+abc-d$

$f(b) = ab^2+b-ab^2-abc+abc-d = b-d$

$f(b)<0$

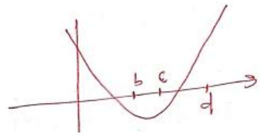
$f(c) = ac^2+(c-abc-ac^2)+abc-d = c-d$

$f(c)<0$

$f(d) = ad^2+d-abd-acd+abc-d$

$f(d) = ad(d-b-c)+abc$

$a>0$



⑤ ⇒ $f(x) = \frac{x^2-2x+3}{x^2-2x-8} = y$

$x^2-2x+3 = yx^2-2xy-8y$

$(y-1)x^2-2x(y-1)-8y-3=0$

$D \geq 0$

$4(y-1)^2+4(y-1)(8y+3) \geq 0$

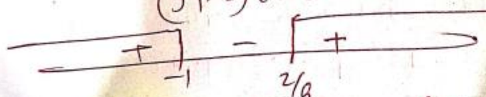
$y^2+1-2y+8y^2+3y-8y-3 \geq 0$

$9y^2+7y-2 \geq 0$

$9y^2+9y-2y-2 \geq 0$

$9y(y+1)-2(y+1) \geq 0$

$(9y-2)(y+1) \geq 0$



$y \in (-\infty, -1] \cup [2/9, \infty)$

(6) If $\sum_{r=1}^{n} T_r = \frac{1}{12} n(n+1)(n+2)$

$S_n = \frac{n(n+1)(n+2)}{12}$

$t_n = S_n - S_{n-1} = \frac{n(n+1)(n+2)}{12} - \frac{(n-1)n(n+1)}{12}$

$= \frac{n(n+1)}{12}(n+2-n+1)$

$t_n = \frac{3}{12} n(n+1)$

$\frac{1}{t_n} = \frac{4}{n(n+1)}$

$\sum_{r=1}^{n} \frac{1}{t_r} = \sum_{r=1}^{n} 4\left(\frac{1}{r(r+1)}\right) = 4\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right)$

$= 4\left(1 - \frac{1}{n+1}\right) = \frac{4n}{n+1}$

7.

$I_n = \int_0^{\pi/4} \tan^n x\, dx$

$I_n = \int_0^{\pi/4} (\tan x)^{n-2}(\sec^2 x - 1)\, dx$

$= \int_0^{\pi/4} (\tan x)^{n-2} \sec^2 x\, dx - \int_0^{\pi/4} (\tan x)^{n-2} dx$

$I_n = \left(\frac{t^{n-1}}{n-1}\right)_0^1 - I_{n-2}$

$I_n + I_{n-2} = \frac{1}{n-1}$

$I_2 + I_4 = \frac{1}{3}$ $I_5 + I_3 = \frac{1}{4}$, $I_6 + I_4 = \frac{1}{5}$

(8) $S_n = \frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + \cdots$

$t_n = \frac{(2n-1)}{n(n+1)(n+2)} = \frac{(2n+4)}{n(n+1)(n+2)} - \frac{5}{n(n+1)(n+2)}$

$= \frac{2}{n(n+1)} - \frac{5}{n(n+1)(n+2)}$

$S_n = \sum t_n = 2\left(1 - \frac{1}{n+1}\right) - \frac{5}{2}\left(\frac{1}{1\cdot2} - \frac{1}{(n+1)(n+2)}\right)$

$= \frac{2n}{n+1} - 5\left(\frac{(n+1)(n+2) - 2}{4(n+1)(n+2)}\right)$

$= \frac{8n(n+2) - 5(n^2+3n+2-2)}{4(n+1)(n+2)}$

$= \frac{n(8n+16-5n-15)}{4(n+1)(n+2)}$

$= \frac{n(3n+1)}{4(n+1)(n+2)}$

(9) If $f(n) = \left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ $\lim_{x\to\infty} f(x)$

$\lim_{x\to\infty}\left(1 + \frac{x^2+5x+3}{x^2+x+2} - 1\right)^x$

$\lim_{x\to\infty}\left(1 + \frac{x^2+5x+3-x^2-x-2}{x^2+x+2}\right)^x$

$\lim_{x\to\infty}\left(1 + \frac{4x+1}{x^2+x+2}\right)^x = e^{\lim_{x\to\infty} \frac{x(4x+1)}{(x^2+x+2)}} = e^4$

(10) If $f(x) = \begin{cases} x^2+4x+5 & x \in \mathbb{Q} \\ 1+x & x \in R-\mathbb{Q} \end{cases}$

is continuous at two points.

$x^2 + 4x + 5 = 1 + x$

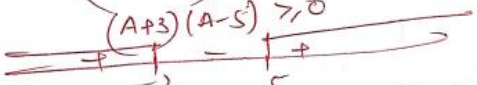
$x^2 + x(4-1) + 4 = 0$

$D \geq 0$

$(4-1)^2 - 16 \geq 0$

$(4-1+4)(4-1-4) \geq 0$

$(4+3)(4-5) \geq 0$



$A \in (-\infty, -3] \cup [5, \infty)$

(11) let $f(x) = \frac{\ln(\pi - \cos^2 x)}{x^2}$ $x \neq 0$ $f(0) = \pi$ if f is conti.

$\lim_{x\to 0} \frac{\ln(\pi - \pi\cos^2 x)}{\pi(1-\cos^2 x)} \pi \frac{\sin^2 x}{x^2} = \pi$

(12) If $f(x) = \sqrt{x + 2\sqrt{2x-4}} + \sqrt{x - 2\sqrt{2x-4}}$

$2x - 4 = t^2$ $x = \frac{t^2+4}{2}$

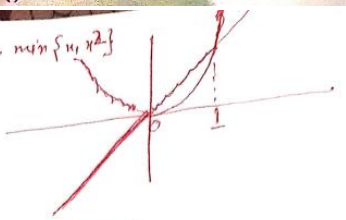
$f(x) = \frac{1}{\sqrt{2}}\left(\left|\sqrt{2x-4}+2\right| + \left|\sqrt{2x-4}-2\right|\right) = \sqrt{\frac{t^2+4}{2} + 2t} + \sqrt{\frac{t^2+4}{2} - 2t}$

$= \frac{1}{\sqrt{2}}\left(\sqrt{t^2+4+4t} + \sqrt{t^2+4-4t}\right)$

$= \frac{1}{\sqrt{2}}\left(\sqrt{(t+2)^2} + \sqrt{(t-2)^2}\right)$

$= \frac{1}{\sqrt{2}}(|t+2| + |t-2|)$

(13) Let $h(x) = \min\{x, x^2\}$



$h(x) = x \quad x > 1$

$h'(x) = 1$

(14) $y = (x^2+1)^{\tan x} = e^{\tan x \log(x^2+1)}$

$y'(x) = \frac{dy}{dx} = (x^2+1)^{\tan x}\left(\tan x \frac{2x}{(x^2+1)} + \log(x^2+1)\sec^2 x\right)$

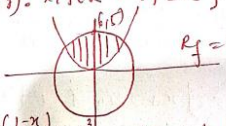
$y'(0) = 0$

(15) we know that $\frac{d^2x}{dy^2} = -\left(\frac{dy}{dx}\right)^3 \times \frac{d^2y}{dx^2}$

$\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3} = -\frac{\frac{d^2x}{dy^2}}{\left(\frac{dy}{dx}\right)^3}$

(16) (d) do yourself

(17) $R = \{(x,y) : x,y \in R,\ x^2+y^2 \le 25\}$ $R' = \{(x,y) : x,y \in R,\ y \ge \frac{4x^2}{5}\}$



$R_f = [0,5]$

(18) (d) $f^2(x) f(\frac{1-x}{1+x}) = x^3$ $x \ne -1, 1$ $\left|\sum f(-2)\right|$ G.I.f

$x \to \frac{1-x}{1+x}$

$f^2\left(\frac{1-x}{1+x}\right) f\left(\frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}}\right) = \left(\frac{1-x}{1+x}\right)^3$

$f^2\left(\frac{1-x}{1+x}\right) f(x) = \left(\frac{1-x}{1+x}\right)^3 \Rightarrow \frac{f(x)}{f\left(\frac{1-x}{1+x}\right)} = \frac{x^3}{\left(\frac{1-x}{1+x}\right)^3}$

$f(x) = x^3 \Rightarrow f(-2) = |[-8]| = 8$

(19) $f(x) = \log_2\left(-\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right)-1\right)$

$x > 0$

$-\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) - 1 > 0$

$\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) + 1 < 0$

$\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) < -1$

$1+\frac{1}{x^{1/4}} > \left(\frac{1}{2}\right)^{-1}$

$1+\frac{1}{x^{1/4}} > 2$

$\frac{1}{x^{1/4}} > 1$

$x^{1/4} < 1 \Rightarrow x < 1$

$x \in (0,1)$

(20) Let $A = \{x \in R : [x+3] + [x+4] \le 3\}$ $B = \{x \in R : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r}\right)^{x-3} < 3^{-3x}\}$

$A = [x] + [x] \le -4$

$[x] \le -2$

$-\infty < x < -1$

$3^x \left(3\left(\frac{1}{10}+\frac{1}{10^2}+\frac{1}{10^3}+\cdots\right)\right)^{x-3} < 3^{-3x}$

$3^x\, 3^{x-3} \times \left(\frac{1/10}{1-1/10}\right)^{x-3} < 3^{-3x}$

$3^x\, 3^{x-3}\, 3^{-2x+6} < 3^{-3x}$

$3^{2x-3-2x+6} < 3^{-3x}$

$3 < -3x$

$-1 > x \Rightarrow x < -1$

(21) A & B as a diameter of circle.

Combined eqn. of OA & OB

$\frac{3x+4y}{k} = 1$

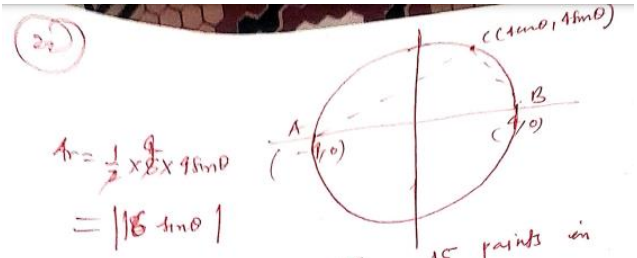
$2x^2+2xy+3y^2 - \left(\frac{3x+4y}{k}\right)^2 = 0$

coeff of x^2 + coeff of $y^2 = 0$

$\left(2-\frac{9}{k^2}\right) + 3 - \frac{36}{k^2} = 0 \Rightarrow 5 = \frac{45}{k^2}$

$k^2 = 9$

(22)



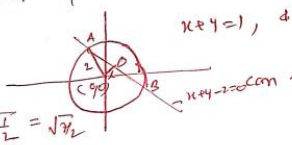
$A_r = \frac{1}{2} \times 8 \times 4\sin\theta$

$= |16\sin\theta|$

$\sin\theta = \frac{1}{16}, \frac{2}{16}, \cdots \frac{15}{16}$ 15 points in each quadrant $\Rightarrow 60+2$ $\because \sin\theta = \pm 1$

$N = 62$

(23) Sum of intercept of length of the chord intercepted by the line $x+y=n$ $n \in M$



$x+y=1$, & $x+y=2$

$n > 2$

$x+y=-2$ can not possible.

$l = \frac{1}{\sqrt{2}}$

$AD = \sqrt{4-\frac{1}{2}} = \sqrt{\frac{7}{2}}$

$AB = 2AD = \sqrt{14}$

$l_2 = \frac{2}{\sqrt{2}} = \sqrt{2}$

$AD = \sqrt{4-2} = \sqrt{2}$

$AB = 2\sqrt{2}$

$= 14+8 = 22$

(24) family of lines $(5x+3y-2)+\lambda(3x-y-4)=0$

& $(x-y+1)+\mu(2x-y-2)=0$

$ax+by-7=0$

$x-y+1=0$

$2x-y-2=0$

$-x+3=0$

$x=3$ $y=4$

$5x+3y-2=0$

$3x-y-4=0$

$9x-3y-12=0$

$14x-14=0$

$x=1$ $y=-1$

$a-b-7=0$

$3a+4b-7=0$

$3b+21+4b-7=0$

$7b=-14$

$b=-2$

$a=5$

$1+b=B$

(25) $ax^2+3xy+by^2-11x+13y+c=0$

Lines are $\perp^r$ $b=-2$

Represent two lines $\Rightarrow abc+2fgh-af^2-bg^2-ch^2=0$

$c=-2$

$|b+2c| = 6$